

Non-Twist Regularization for Deformation Estimation

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Abstract

In addition to seeking geometric correspondence between the inputs, a legitimate image registration algorithm should also keep the estimated transformation meaningful or regular. In this paper, we present a mathematically sound formulation that explicitly controls the deformation to keep each grid in a meaningful shape over the entire geometric matching procedure. The deformation regularity conditions are enforced by maintaining all the moving neighbors as non-twist grids. In contrast to similar work, we differentiate and formulate the convex and concave update cases under an efficient and straightforward point-line/surface orientation framework, and use equality constraints to guarantee grid regularity and prevent folding. Experiments on MR images are presented to show the improvements made by our model over the popular Demon's [1] and DCT-based [2] registration algorithms.

1 Introduction

Image registration is one of the most fundamental problems in image analysis. It's often the prerequisite step for many other analysis tasks to be carried out, especially in medical applications where images from different modalities are utilized to provide complementary information for diagnosis and treatment. Often phrased as an optimization procedure, non-rigid image registration is known as an inherently ill-posed problem with infinitely many solutions for a pair of inputs. Regularization is required in order to produce meaningful matching and integrate user knowledge into the problem formulation.

Two types of regularizations have been extensively researched in the literature. One direction is focused on imposing smoothness to the estimated deformation field. Many works use regularizer to penalize deviation from some measure of smoothness, either in the estimated deformation field [3] or its update [4]. Smoothness can also be achieved through a diffusion procedure, where Gaussian convolution kernels are commonly utilized, as in the Demon's algorithm [1]. The second type of regularization is to ensure topology preservation and invertibility, so that every point in one image has a corresponding point in the other. Such guaranteed correspondence is particularly important in various longitudinal studies where the output of the registration, i.e. the deformation field, is analyzed further to draw quantitative conclusions that can potentially be used as the basis for prediction and diagnosis.

Limiting the Jacobian determinants $J_T(x)$ to a range has been a popular approach to enforce topology regularity. In order to model incompressible organ tissues, Rohlfing *et*

al. [10] use $\int \log(J_T)dx$ as an additional regularization term to penalize local deviations of $J_T(x)$ from unity, which in turn, penalizes local tissue expansion and compression. Blended together with a smoothness constraint within the overall objective function, the incompressibility property is favored, but not guaranteed. In [11], Haber *et al.* set upper and lower bounds for the Jacobians during the registration procedure. A log-barrier function scheme is employed to convert the constraints to an unconstrained optimization problem, where Gauss-Newton approximation is applied to solve the system, which contains a large number of unknowns. In [12], topology preservation is achieved through a hard constraint at several intermediate steps of a deformable registration procedure or after the registration is done. Sdika [13] uses B-spline as the global smoothness constraint. The topology regularity is imposed through the positivity of the Jacobians as well as the bounded Jacobian derivatives. These three works and their variants have one thing in common: the enforced regularity cases specified by the Jacobian constraints are all convex, where the concave cases, allowed in many applications, have been ruled out. Detailed explanation of the convex/concave regularity cases will be given in section 2.

The diffeomorphic Demon’s algorithm [14] uses a fast exponential to project the deformation fields into a Lie group, which in nature guarantees the smoothness and invertibility of the elements. Similar to the original Demon’s algorithm, a diffusion-like regularization is applied to the resulting deformation field (optional to the deformation update as well), which could lead to globally oversmoothing effects, and fail to produce high-precision registration.

1.1 Our Proposed Work

In this paper, we propose a regularity guaranteed deformation estimation through a convenient and efficient point-line/surface orientation perspective. We identify two types of regularity ensured conditions: convex and concave Non-Twist updates, and the corresponding mathematical constraints are proposed and analyzed.

We integrate our Non-Twist components into two sum of squared differences (SSD) based registration schemes, Demon’s and the Discrete cosine transform (DCT)-based algorithm [15] in SPM. Comparisons with the original algorithms are presented to show the effects and improvements made by our approach.

2 Non-twist Regularization

Topology regularity, or globally one-to-one property, will be violated if grid corners flip their relative positions during the registration procedure. Shown in figure 1 is a 2D grid and the associated deformation scenarios. Let A be the point of interest, and $ABCD$ be the grid to study. After each spatial update, A might end up in one of the three destination areas in figure 1 (a), marked with gray, orange, and blue colors, respectively. The corresponding resultant grids are shown in figure 1 (b), (c) and (d). Because it satisfies the positivity of the Jacobian criterion, scenario (b) is usually taken as a topology/regularity preservation case, and we call it the *convex* case in this paper, as it maintains the convexity of the starting grid $ABCD$. Scenario (c), which we call the *concave* case, is regarded as illegal and ruled out by most aforementioned works [10, 11, 12], even though it is acceptable in reality as no twisting has happened. Case (d) is indeed twisting the grids and it should be prevented from happening.

Identifying the three cases (convex and concave, and twisting) can be easily performed through the point-line/surface test [16] commonly used in the computational geometry community. The *convex* case requires a combination of three positive orientations: A is on the

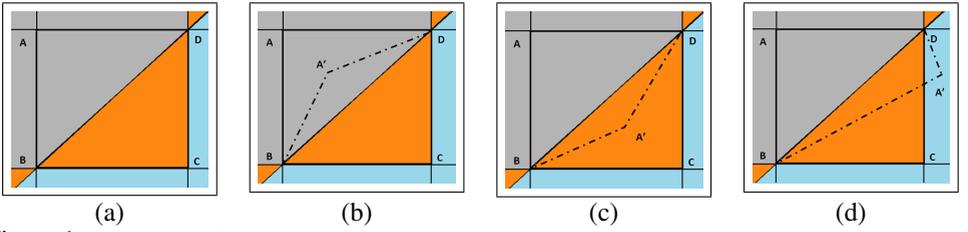


Figure 1: Illustration of possible grid update scenarios. (a) starting grid; (b) convex case; (c) concave case; (d) twisting case, which should be avoided.

left side of \overrightarrow{BD} , left side of \overrightarrow{BC} and left side of \overrightarrow{CD} . Converting to the matrix format, the following three constraints have to be held to ensure the *convex* case.

$$\begin{vmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_d & y_d & 1 \end{vmatrix} > 0, \quad \begin{vmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_c & y_c & 1 \end{vmatrix} > 0, \quad \begin{vmatrix} x_a & y_a & 1 \\ x_c & y_c & 1 \\ x_d & y_d & 1 \end{vmatrix} > 0 \quad (1)$$

Combining the convex and concave cases together (the gray and orange areas in fig 1, the overall accepted update scenarios can be translated to a Boolean predicate: (A is on the left side of \overrightarrow{BD}) **OR** (A is on the left side of \overrightarrow{BC}) **AND** (A is on the left side of \overrightarrow{CD})). This determinant formulation can be easily generalized for 3D and N-D inputs.

2.1 Integration of the Non-Twist Regularization into Registration

Our Non-Twist regularization can be easily integrated into various registration algorithms. In this paper, we choose Demon’s algorithm and DCT-based [10] as the testbed to show the effectiveness of the additional term. Justification of using these two algorithms lies in the fact that they belong to different algorithm categories (non-parametric and parametric), and both have been widely used across the neuroimage community.

Like many other non-rigid registration algorithms, Demon’s and DCT are also formulated as the optimization of an objective function to determine a deformation field u that minimizes the difference between the reference image R and floating image F . Demon’s uses SSD as the similarity metric, and the squared gradient of the transformation field as smoothness regularization. Given the transformation field S , compute a correspondence update field u by minimizing E

$$E(u) = |F - M \cdot (S + u)|^2 + \sigma^2 |u|^2 \quad (2)$$

where σ is a constant for intensity and transformation uncertainty.

DCT-algorithm also uses SSD as the similarity metric. Unlike in Demon’s, the smoothness property in DCT, however, is fulfilled through a linear combination of lower-frequency components of the DCT. The deformation field can be obtained by solving a small set of coefficients controlling the basis functions. For more details, we refer readers to [10].

Integrating our Non-Twist constraint with Demon’s and DCT is straightforward. Taking the convex case as an example, the new optimization objective is simply

$$\begin{aligned} \text{Minimize:} \quad & E(u) \\ \text{Subject to:} \quad & (A \uparrow BD) \text{ AND } (A \uparrow BC) \text{ AND } (A \uparrow CD) \end{aligned}$$

where $A \uparrow BD$ denotes the requirement that A lies on the left side of \overrightarrow{BD} .

Numerical Solution: To implement this optimization with our proposed constraint, a large-scale nonlinear algorithm using interior-point based sequential quadratic programming (SQP) [9] is adopted and adjusted specifically. Due to its use of a trust region framework that allows for the direct use of second derivatives and the inaccurate solution of sub-problems, this optimization algorithm is comparably efficient in dealing with the dense deformation field. Since the whole problem is large and indefinite, we directly take use of the currently available function (Fmincon) in MATLAB Optimization Toolbox.

3 Experimental Results

We assess the effectiveness of our non-twist regularizer through two paired algorithms: Demon’s vs. Demon’s + Non-Twist and DCT vs. DCT + Non-Twist.

Demon’s algorithm uses Gaussian convolution to impose smoothness on the estimated deformation field, where the level of smoothness is controlled by the σ of the Gaussian filter. Since the smoothness is evenly applied throughout the image domain, this penalty approach with a moderate Gaussian σ does not prevent local twists. On the other hand, it is well-known that increasing σ generally leads to over-smoothing effect, which results in deteriorated precision in the registration results.

The reference image (as in fig 2.a) used in our experiment is a synthetic MRI slice, 256×256 dimension and $1 \times 1 \text{ mm}^2$ resolution, obtained from BrainWeb simulation [9]. It is deformed into fig 2.b with a known sinusoidal transformation shown in fig 2.c. The registration result using the original Demon’s algorithm is depicted in fig 2.d. The σ of the Gaussian convolution filter in this experiment was set to 6. As evident, local irregularities emerge in the upper left and bottom right brain boundary areas where no sufficient intensity detail exists to enforce the smoothness property. The estimated deformation field shown in fig 2.e comes from the new Demon’s algorithm with our Non-Twist regularizer (concave version in this particular example). Obviously, all the irregularities have been driven away by the additional regularization component. The same input image pair has been used for the comparison of DCT alone vs. DCT + Non-Twist. Results are shown in fig 2.f and fig 2.g, respectively. Same number of coefficients, 32×32 , are used in the experiments. It should be noted that DCT alone has built-in global smoothness control stemmed from its parametric deformation setup. However, the original method has no guaranteed mechanism to avoid local twists. The improvements with the added Non-Twist are obvious – fig 2.g has much smoother grids and no visible local foldings.

4 Conclusion and Discussion

Maintaining regularity and topology legitimacy is an important issue in image registration. Most of the previously published works are focused on using the Jacobians as the basis to specify regularity constraints, and most of the works only allow convex grid deformations. We identify the concave case and formulate the new constraints under a straightforward point-line/surface orientation perspective. Our experimental results indicate the added constraints can greatly improve the overall registration performance, especially in terms of smoothness and regularity.

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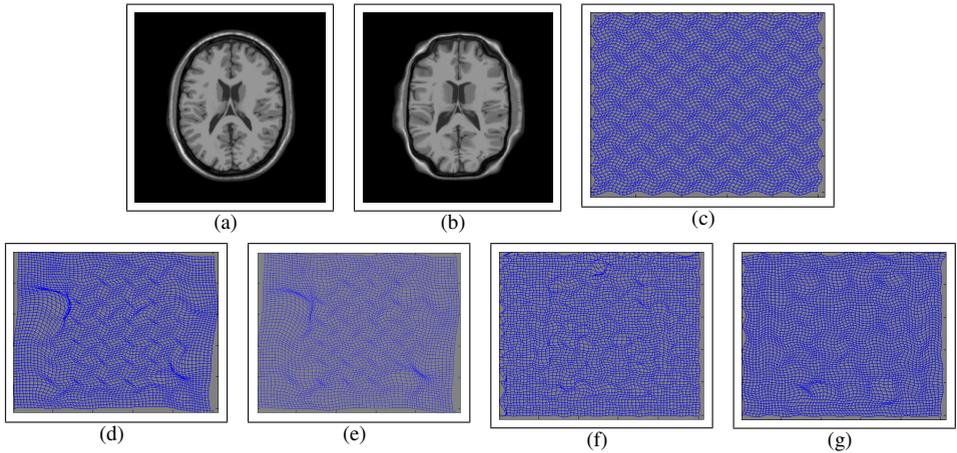


Figure 2: Registration results with and without Non-Twist regularization. (a)reference image; (b) floating image; (c) groundtruth deformation field. (d) estimated field using Demon's; (e) using Demon's + Non-Twist. (f) estimated field using DCT alone; (g) using DCT + Non-Twist constraint.

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